DBDM

TD n°2

Exercise 2.1.

(Natural) join $\bowtie$ is associative and comutative.

Exercise 2.2.

It's the cartesian product.

Exercise 2.3.

1) We suppose $\{a_1, \dots, a_n\} \cap \{b_1, \dots, b_n\} = \emptyset$ and $\forall i \neq j,\ a_i \neq a_j,\ b_i \neq b_j$

$$R \underset{\substack{a_1 = b_1 \\ \vdots \\ a_n = b_n}}{\bowtie} S = \sigma_{a_1 = b_1}\left(\sigma_{a_2 = b_2}\left(\cdots \sigma_{a_n = b_n}(R \times S) \dots \right)\right)$$

$$\pi_{(A_R \cup A_S) \setminus \{a_1, \dots, a_n\}} R \underset{\substack{a_1 = b_1 \\ \vdots \\ a_n = b_n}}{\bowtie} S = \rho_f(R) \bowtie S$$

$$f : A_R \to A_R'$$
$$a_i \mapsto b_i$$
$$\{a_1, \dots, a_n\} \not\ni a \mapsto a$$

2) Define $T = A_R \cap A_S$, and

$$f : A_S \to Att$$
$$T \not\ni a \mapsto a$$
$$T \ni a \mapsto a'$$

fresh attribute

$$R \bowtie S = \pi_{A_R \cup A_S}\left(R \underset{a = a'}{\bowtie} S\right)$$

Exercise 2.4.

$$\sigma_{travel \neq travel'}\left(\pi_{travel, travel'}\left(usedFor \bowtie \rho_f(usedFor)\right)\right)$$

$$f : train \mapsto train'$$
$$date \mapsto date'$$
$$travel \mapsto travel'$$

$$\sigma_{a \neq b}(q) = q \setminus \sigma_{a = b}(q)$$

Exercise 2.5.

$$\sigma_{j=a}(R) = R \bowtie S.$$
$$= \{u \in D^{A_R} \mid \underbrace{u(j)=a}_{u \in S}\}.$$

Exercise 2.6.

1. $R_1, \dots, R_m$ joins completely
   iff $\forall t \in R_i,\ \exists t' \in R_1 \bowtie \cdots \bowtie R_m,\ t'[A_{R_i}] = t$
   iff $\forall t \in R_i,\ t \in \pi_{A_{R_i}}(R_1 \bowtie \cdots \bowtie R_m)$
   iff* $R_i = \pi_{A_{R_i}}(R_1 \bowtie \cdots \bowtie R_m)$

   $*$: $\Rightarrow$ $R_i \subseteq \pi_{A_{R_i}}(R_1 \bowtie \cdots \bowtie R_m)$ by assumption
   $R_i \supseteq \pi_{A_{R_i}}(R_1 \bowtie \cdots \bowtie R_m)$ by def.

2. We simply have to prove that $R_i = \pi_{S_i}(R_1 \bowtie \cdots \bowtie R_m)$.
   We have that $R_1 \bowtie \cdots \bowtie R_m = \pi_{\bigcup_i S_i}(R)$ and so
   $$\pi_{S_i}(R_1 \bowtie \cdots \bowtie R_m) = \pi_{S_i}(\pi_{\bigcup_i S_i}(R)) = \pi_{S_i}(R) = R_i.$$

Exercise 2.7.

We have to have $A \cap B$ as primary keys of R.
It is not sufficient.

Exercise 2.8.

$$(R \bowtie S) \div S = (R \times S) \div S = S$$

TD n°3

Exercise 1.1.

Define $D = \{1, 2, 5, \text{Patrick}, \text{Samia}, \text{Mitsuko}, 12, \dots\}$,

$$\text{Client}^I = \{(1, \text{Patrick}, 12), (2, \text{Samia}, 88), (5, \text{Mitsuko}, 42)\}$$

$$\text{Reservation}^I = \{\dots\}$$

Exercise 1.2.

1) $\exists\, age,\ \text{Client}(\underline{\underline{id}}, \text{"Patrick"}, age) \vee \text{Client}(\underline{\underline{id}}, \text{"Robert"}, age)$

2) $\forall client, (\exists\, name, \exists\, age,\ \text{Client}(client, name, age) \Rightarrow \exists\, arival,$
$\text{Reservation}(client, \underline{\underline{room}}, arival))$

$$\text{where } (\varphi \Rightarrow \psi) := \psi \vee (\neg \varphi).$$

3) $\forall id, \forall age, \neg \text{Client}(id, \underline{\underline{name}}, age)$

but name could be something other than a name.

Exercise 1.3.

1. Yes by Logique (L3)
2. No! Define $\varphi_n := \exists\, a_1 \dots a_n,\ \bigwedge_{i \neq j} a_i \neq a_j$

"there is at least n distinct elements"

with $\Gamma = \{\varphi_n \mid n \in \mathbb{N}\}$.

Γ is finitely satisfiable by finite models, but Γ does not have finite models.

Thus it is not compact.

Exercise 1.4.

We use a relation $R = - < -$ with the formula

$$\forall x, \neg (x < x)$$

$$\wedge$$

$$\forall x y z, x < y \wedge y < z \Rightarrow x < z$$

$$\wedge$$

$$\forall x \, \exists y \quad x < y .$$

Exercise 1.5.

1. Algorithm $Eval(\varphi, \lambda)$:

if $\varphi = \bot$ then return false
if $\varphi = \top$ then return true
if $\varphi = \varphi_1 \wedge \varphi_2$ then return $Eval(\varphi_1, \lambda)$ && $Eval(\varphi_2, \lambda')$
if $\varphi = \varphi_1 \vee \varphi_2$ then return $Eval(\varphi_1, \lambda)$ || $Eval(\varphi_2, \lambda')$
if $\varphi = \forall x, \varphi'$ then
  for all $v \in D$ do
    if $Eval(\varphi', \lambda[x := v]) =$ false then return false
  return true
if $\varphi = \exists x, \varphi'$ then
  for all $v \in D$ do
    if $Eval(\varphi', \lambda[x := v])$ then return true
  return false
if $\varphi = R(\tau_1, \dots, \tau_n)$ then $R^I(\lambda(\tau_1), \dots, \lambda(\tau_n))$

space
$O(|\varphi| \log |D|)$
time
$O(|D|^{|\varphi|})$

2. ?

Exercise 1.6.

	All models	Finite models
Valid	RE by proof search	$\neg$RE
Satisfiable	$\neg$RE	RE by finite model