Assignment #1.

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M1 QCS

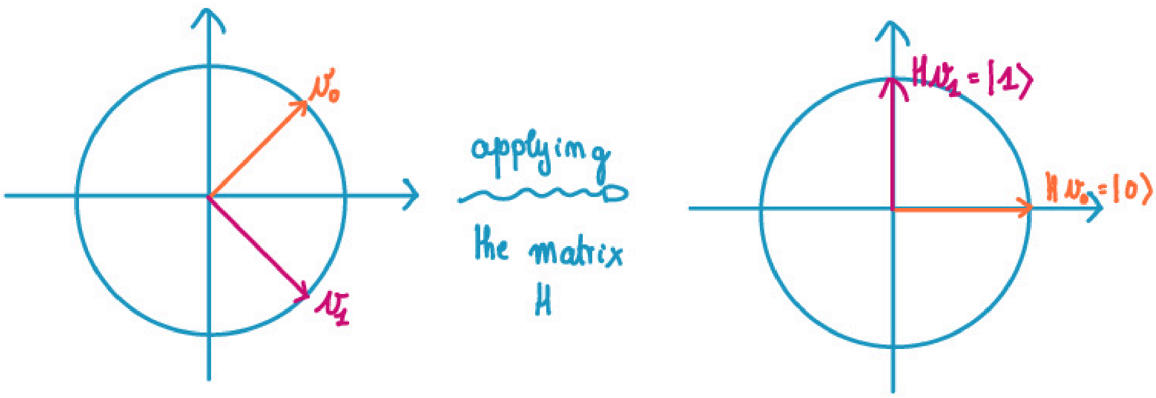
Q1. To know whether system X is in state v_0 or v_1 we can apply the Hadamard matrix:

$$H = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}.$$

We have that $HH = \mathbb{1}_2$, $H|0\rangle = v_0$, and $H|1\rangle = v_1$. Thus, applying the Hadamard matrix again, we have that:

$$H v_0 = |0\rangle \quad \text{and} \quad H v_1 = |1\rangle.$$

We can distinguish Hv_0 from Hv_1 by measuring.



Q2. We will prove that $|\varphi\rangle := \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$ is entangled with a proof by contradiction. Suppose $|\varphi\rangle = u \otimes v$ for some qubits u and v, with $u = \begin{pmatrix} a \\ b \end{pmatrix}$ and $v = \begin{pmatrix} c \\ d \end{pmatrix}$.

Then, we have that

$$u \otimes v = \begin{pmatrix} a\begin{pmatrix} c \\ d \end{pmatrix} \\ b\begin{pmatrix} c \\ d \end{pmatrix} \end{pmatrix} = \begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 0 \\ 1/\sqrt{2} \end{pmatrix}.$$

In particular, we have that $ad = 0$, so either $a = 0$ or $d = 0$.

If $a = 0$ then $ac \neq 1/\sqrt{2}$, absurd!

If $d = 0$ then $bd \neq 1/\sqrt{2}$, absurd!

We can conclude that $\begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$ is entangled.

END of ASSIGNMENT #1