Assignment #2.

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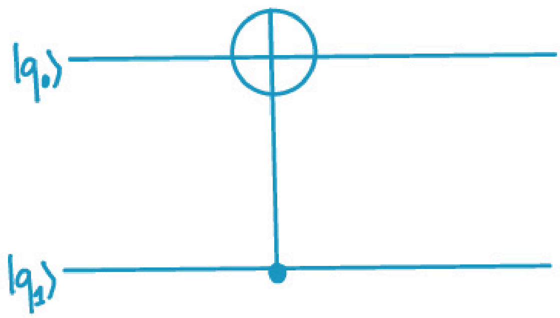
Q1. We can compute explicitely the unitary matrix of the quantum circuit:

$$\frac{1}{4}\left(\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\otimes\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\right)\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix}\left(\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\otimes\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\right)$$

$$= \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\end{pmatrix}$$

We can recognize the unitary matrix of the following quantum circuit:

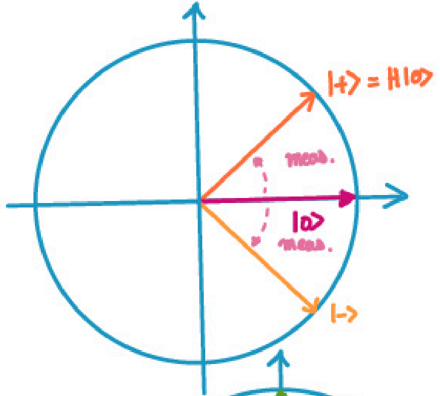


which is a lot simpler.

Q2. We get 1 qubit set at $|0\rangle$ from the machine. Then, we measure in the $\{|+\rangle, |-\rangle\}$ basis and measure in the $\{|0\rangle, |1\rangle\}$ basis.

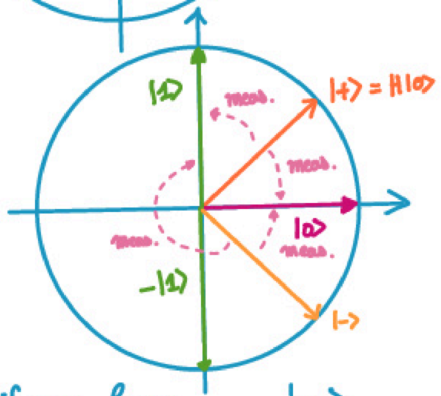
The result of the first measure will be:

- $|+\rangle$ with probability $1/2$;
- $|-\rangle$ with probability $1/2$.



Then, for the second measure, we have:

- $|1\rangle$ with probability $1/2$;
- $|0\rangle$ with probability $1/2$.



If we have a $|1\rangle$, we are done, but if we have a $|0\rangle$, we simply repeat the procedure again and again.

The number of iterations follows a geometric law $\mathcal{G}(1/2)$, thus we can "expect" to have to do $\mathbb{E}[X] = 2$ iterations to obtain a $|1\rangle$.
Repeating 7 times the procedure will give us a $|1\rangle$ with more than 99% probability.

A more complex procedure could give us better odds.

End of Assignment #2