Assignment # 3.

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Q1. No! Let $\mathcal{B} := \left\{ \underbrace{\frac{|0\rangle + i|1\rangle}{\sqrt{2}}}_{|b_0\rangle}, \underbrace{\frac{|0\rangle - |1\rangle}{\sqrt{2}}}_{|b_1\rangle = |-\rangle} \right\}$.

Measuring the first qubit of $|\Phi^+\rangle$ in $\mathcal{B}$ and getting $|b_0\rangle$ gives us the following state vector:

$$|\tilde{\Phi}^+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) = 0 \times |b_0\rangle + 1 \times |b_1\rangle.$$

However, measuring the last qubit, we are guaranteed to obtain $|b_1\rangle$ and not $|b_0\rangle$.

Thus, the statement is false.

By the symmetry of the state vector, measuring the 2^{nd} qubit and getting $|b_0\rangle$ also gives $|b_1\rangle$ as a state vector.

Q2 Define $|b_0\rangle = \alpha|0\rangle + \beta|1\rangle$. We measure the first qubit of the state vector $|\Phi^-\rangle$ and get the following state vector:

$$|\tilde{\Phi}^-\rangle = \frac{1}{\sqrt{2}}\left(\bar{\alpha}\,|1\rangle - \bar{\beta}\,|0\rangle\right).$$

Measuring the other qubit, we have the following probability of getting $|b_0\rangle$:

$$\Pr[\tilde{\Phi}^- \text{ measured as } |b_0\rangle] = |\langle b_0|\tilde{\Phi}^-\rangle|^2$$
$$= \left|\frac{\bar{\alpha}\bar{\beta} - \bar{\alpha}\bar{\beta}}{\sqrt{2}}\right|^2$$
$$= 0.$$

Thus, we always measure $|b_1\rangle$ at the second measurement.

Conversely, by the symmetry of the system (except a "$-$" sign, which won't add any problem as probability of measurment uses $|\cdot|$), we get the same result if we measure the 2nd qubit and get a $|b_1\rangle$, and then measure the 1st qubit.

END of ASSIGNMENT #3